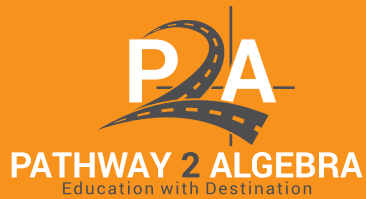


Pathway2Careers Algebra I





Pathway2 Algebra I



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Pathway2Careers Algebra I

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LESSON 2.2

Solving Linear Equations with a Variable on One Side



CAREER SPOTLIGHT: Veterinarian

Occupation Description

Veterinarians care for the health of animals and work to protect public health. They diagnose, treat, and research medical conditions and diseases of pets, livestock, and other animals.

Veterinarians treat the injuries and illnesses of pets and other animals with a variety of medical equipment, including surgical tools and x-ray and ultrasound machines. They provide treatment for animals that is similar to the services a physician provides to humans.

Education

Veterinarians must complete a Doctor of Veterinary Medicine (DVM or VMD) degree at an accredited college of veterinary medicine. A veterinary medicine program generally takes 4 years to complete and includes classroom, laboratory, and clinical components.

Potential Employers

The largest employers of veterinarians are as follows:

Veterinary services	78%
Self-employed workers	14%
Government	3%
Social advocacy organizations	1%
Educational services; state, local, and private	1%

Watch a video about veterinarians:

<https://cdn.careeronestop.org/OccVids/OccupationVideos/29-1131.00.mp4>

Career Cluster

Health Science

Career Pathway

Therapeutic Services

Career Outlook

- Salary Projections:
Low-End Salary, \$58,080
Median Salary, \$95,460
High-End Salary, \$160,780
- Jobs in 2018: 84,500
- Job Projections for 2028:
100,100 (increase of 18%)

Algebra Concepts

- Write linear equations in one variable.
- Use linear equations in one variable to solve real-world problems.

Is this a good career for me?

Veterinarians:

- Examine animals to assess their health and diagnose problems
- Perform surgery on animals
- Test for and vaccinate against diseases
- Advise animal owners about general care, medical conditions, and treatments

Lesson Objective

In this lesson, you will look at how a veterinarian writes and solves linear equations to promote the health of animal patients.

1 Step Into the Career: Solving One-Step Equations

A veterinarian is prescribing medicine to a dog patient to fight infection. The suggested dosage is 1 milligram per kilogram of the dog's mass. How many pounds does the dog weigh if the dosage is 8 milligrams? Use $1 \text{ lb} = 0.454 \text{ kg}$.



Devise a Plan

Step 1: Find the dog's mass in kilograms given the dosage.

Step 2: Write an equation to convert between kilograms and pounds.

Step 3: Solve for the dog's weight in pounds.

Walk Through the Solution

Step 1: To find the mass of the dog in kilograms, divide the dosage by 1 milligram per kilogram.

$$8 \text{ mg} \div 1 \text{ mg/kg} = 8 \text{ kg}$$

If the suggested dosage is 1 milligram per kilogram, then the mass of the dog is 8 kilograms.

Step 2: Since $1 \text{ lb} = 0.454 \text{ kg}$, an equation relating the dog's weight in pounds x with the dog's mass in kilograms y is $y = 0.454x$.

Step 3: Let $y = 8$, and solve for x .

$$8 = 0.454x \quad \text{Substitute 8 for } y.$$

$$\frac{8}{0.454} = \frac{0.454x}{0.454} \quad \text{Divide each side by 0.454.}$$

$$17.6 \approx x \quad \text{Simplify.}$$

So, the dog weighs about 18 pounds.

On the Job: Apply Solving One-Step Equations

1. A cat needs a prescription to treat a breathing problem. The suggested dosage is 50 milligrams per kilogram of the cat's mass. Write and solve an equation to find the mass of a cat in kilograms that needs a dosage of 175 milligrams.
 - a. Write an equation relating the cat's weight in kilograms x with the dosage in milligrams y .
 - b. Solve the equation for x to find the mass of the cat.



2 Step Into the Career: Writing and Solving Two-Step Equations

A veterinarian is treating a dog that has suffered trauma and lost blood. He estimates that the dog has about 300 milliliters of red blood cells. The veterinarian wants to give the dog a blood transfusion with whole blood that is 50% red blood cells. How much blood should the veterinarian transfuse into the dog for it to have 500 milliliters of red blood cells?



Devise a Plan

Step 1: Write an equation for the volume of red blood cells the dog will have after a transfusion.

Step 2: Substitute the known values from the problem.

Step 3: Solve for the volume of blood needed to transfuse into the dog.

Walk Through the Solution

Step 1: Let x be the volume of blood transfused and y be the volume of red blood cells in the dog.

$$\begin{array}{rccccccc} \text{volume of red blood cells} & & & & \text{50\% of volume of} & & \text{volume of red blood cells} \\ \text{in dog after transfusion} & = & & \text{transfused blood} & + & & \text{in dog before transfusion} \\ y & = & & 0.5x & + & & 300 \end{array}$$

Step 2: The veterinarian wants the dog to have 500 milliliters of red blood cells, so substitute 500 for y in the equation.

$$500 = 0.5x + 300$$

Step 3: Solve for x .

$500 = 0.5x + 300$	Write the equation.
$500 - 300 = 0.5x + 300 - 300$	Subtract 300 from each side.
$200 = 0.5x$	Simplify.
$2(200) = 2(0.5x)$	Multiply each side by 2.
$400 = x$	Simplify.

The veterinarian should transfuse 400 milliliters of blood into the dog.

On the Job: Apply Writing and Solving Two-Step Equations

2. A veterinarian is treating a cat. She estimates that the cat has about 9 milliliters of red blood cells. The veterinarian wants to give the cat a blood transfusion with blood that is 60% red blood cells. How much blood should the veterinarian transfuse into the cat for it to have 15 milliliters of red blood cells?
- Write an equation relating the volume of the transfusion in milliliters x with the volume of blood in the cat after the transfusion in milliliters y .
 - Solve the equation for x . Interpret the solution.



3 Step Into the Career: Using the Distributive Property

A veterinarian has 400 milliliters of a solution that is 10% medicine. Her patient needs 55 milliliters of the actual medicine. How much more 10% solution does she need for her patient?



Devise a Plan

Step 1: Write an equation for the volume of medicine needed y and the amount of additional solution needed x .

Step 2: Substitute the known values from the problem.

Step 3: Solve for the volume of medicine needed to add to the solution.

Walk Through the Solution

Step 1: Let x be the additional volume of solution needed and y be the total volume of medicine needed.

$$\begin{array}{rcll} \text{volume of} & = & 10\% \text{ of} & \left(\begin{array}{l} \text{volume of} \\ \text{solution to start} \end{array} + \begin{array}{l} \text{additional volume of} \\ \text{solution needed} \end{array} \right) \\ \text{medicine needed} & & & \\ y & = & 0.1(& 400 + x) \end{array}$$

Step 2: The veterinarian wants the patient to have 55 milliliters of actual medicine cells, so substitute 55 for y .

$$55 = 0.1(400 + x)$$

Step 3: Solve for x .

$$55 = 0.1(400 + x)$$

$$55 = 40 + 0.1x$$

Use the distributive property.

$$55 - 40 = 40 - 40 + 0.1x$$

Subtract 40 from each side.

$$15 = 0.1x$$

Simplify.

$$\frac{15}{0.1} = \frac{0.1x}{0.1}$$

Divide each side by 0.1.

$$150 = x$$

Simplify.

The veterinarian should add 150 milliliters of solution with 10% medicine.

On the Job: Apply Using the Distributive Property

3. A litter of 6 cats are at an animal hospital. They each consume 15 milliliters of formula per feeding. Each cat in the litter has already had 4 feedings today. How many more feedings can the cats be fed from a container that had 450 milliliters at the start of the day?
- Write an equation relating the volume of the formula in milliliters y with the number of additional feedings per kitten x .
 - Solve the equation for x . Interpret the solution.

Career Spotlight: Practice

4. A veterinarian has a dog patient who has ingested human pain medication that is toxic to dogs. To treat the dog for toxicity, the dog needs a medicine with a dosage that is 140 milligrams per kilogram. What is the mass of a dog in kilograms that needs a 2100 milligram dose?

Devise a Plan

Step 1: Write an equation relating the weight of the dog and the dosage.

Step 2: _____ ? _____

Step 3: _____ ? _____



5. A veterinarian prescribes medication for a cat. The suggested dosage is 1 milligram per 8 kilograms of the cat's mass. Determine the recommended dosage for a cat that has a mass of 6.8 kilograms.
- Write an equation relating the cat's mass in kilograms and the dosage in milligrams.
 - Solve for the dosage in milligrams. Round to the nearest hundredth of a milligram.
6. A veterinarian has 300 milliliters of a 5% solution of the medicine needed but wants a 4% solution. Determine how much pure water needs to be added to make a 4% solution.
- Find the amount of medicine in 300 milliliters of a 5% solution.
 - Write an expression for the amount of medicine in the 4% solution. Let x be the amount of water added.
 - Write and solve an equation relating the amount of medicine in the 5% solution and in the 4% solution. How much pure water does the veterinarian need to add?

7. A veterinarian is treating a horse. She estimates that the horse has about 6 liters of red blood cells. The veterinarian wants to give the horse a blood transfusion with blood that is 40% red blood cells. How much blood should the veterinarian transfuse into the horse for it to have 8 liters of red blood cells? Explain.
8. A veterinarian recommends that the nutritional intake for a 65-pound dog is 1240 calories per day. Suppose a 65-pound dog gets 2 meals a day and treats twice a day. If each meal has 555 calories, how many calories should be in each serving of treats?



Career Spotlight: Check

9. A small dog is suffering moderate pain from a broken leg. The veterinarian suggests a pain medication with a dosage of 3 milligrams per kilogram for moderate pain. The medication comes in 50 milligram tablets to be given every 4 hours as necessary. If the dog has a mass of 15 kilograms, determine how the veterinarian should write the prescription.

Select the answer from each box that makes the sentence true.

First, the veterinarian should find the suggested dosage for the dog by writing the

equation
a. $y = 3x$
b. $y = 3 + x$
c. $y = \frac{x}{3}$
, where x is the mass of the dog and y is the proper dosage.

Then the veterinarian should substitute 15 for
a. x
b. y
 in the equation and solve.

The veterinarian should suggest that the dog take
a. one tablet
b. one and one-half tablets
c. two tablets
 every 4 hours.

10. A rabbit is prescribed medication that comes in 4 dosages. The appropriate dosage is based on 0.25 milligram of medicine for every kilogram of the animal. Which dosage should the veterinarian prescribe for a 1.9 kilogram rabbit?

A. 0.5 mg
B. 1 mg
C. 1.5 mg
D. 2 mg



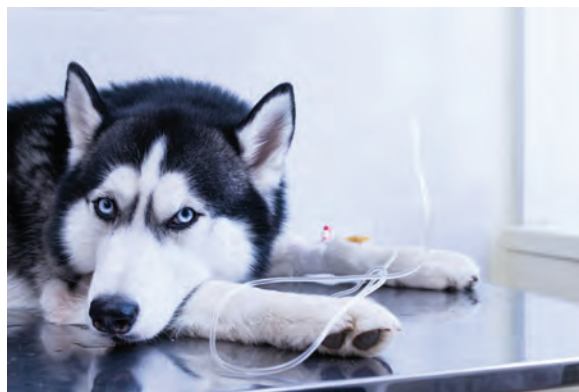
11. A veterinarian makes recommendations for how much cats should eat each day. To maintain a constant healthy weight of x pounds, a cat needs to eat about 56 calories more than 20 calories per pound.

Select all the true statements.

- a. The number of calories a cat should eat each day is represented by $y = 20x + 56$.
b. The number of calories a cat should eat each day is represented by $y = 20(x + 56)$.
c. The number of calories a cat should eat each day is represented by $y = 56(x + 20)$.
d. A healthy 7.5-pound cat should eat about 206 calories per day.
e. A healthy 8.2-pound cat should eat about 479 calories per day.
f. A healthy 10.1-pound cat should eat about 258 calories per day.

12. A veterinarian at an animal hospital is treating a dog who needs IV fluids. The amount of IV fluids the dog needs per day in the hospital is 70 milliliters more than 30 milliliters per kilogram.

- a. Write an equation for the volume of IV fluids the dog needs in one day. Let x be the mass of the dog in kilograms and y be the amount of IV fluids in milliliters.
b. Are 1150 milliliters of IV fluids enough for a 40-kilogram dog? Explain.



13. A veterinarian needs 100 milliliters of a 2% solution of medicine. She has a large bottle of distilled water (a 0% solution) and a large bottle that has a 5% solution of the medicine. How much does she need from each bottle?
- A. 40 mL of distilled water and 60 mL of 5% solution
 B. 50 mL of distilled water and 50 mL of 5% solution
 C. 60 mL of distilled water and 40 mL of 5% solution
 D. 80 mL of distilled water and 20 mL of 5% solution
14. Every ounce of unsweetened chocolate has 8.8 milligrams of toxins that can harm dogs. If a 48-kilogram dog consumes at least 20 milligrams of the toxins in chocolate, it needs emergency medical attention. What is the smallest amount of unsweetened chocolate that will require an emergency visit to a veterinarian for a 48-kilogram dog? Round to the nearest tenth of an ounce.
15. A veterinarian uses an anesthetic to sedate animals to perform dentistry on them. The recommended dosage is 4.5 milligrams per pound. Determine if a 30-milligram dose of anesthetic is sufficient for a cat that weighs 6.5 pounds.

Select the answer from each box that makes the sentence true.

The recommended dosage in milligrams y for a cat that weighs x pounds

is

a. $y = 30 + 6.5x$
b. $y = 4.5x$
c. $y = 30 + 4.5x$

 . Substitute

a. 4.5
b. 6.5
c. 30

 for y , and solve for x . The 30-milligram

dose

a. is
b. is not

 sufficient because the solution

a. $x \approx 4.615$
b. $x \approx 6.667$

 pounds

is

a. more than
b. less than
c. the same as

 the weight of the cat.

16. A veterinarian has suggested that a kitten should gain about 3 ounces per week for the first few months after it turns 6 weeks old. Let x be the age of the cat in weeks and y be the weight gained in ounces by the kitten since it turned 6 weeks old. Which equation shows how much weight the kitten should gain?
- A. $y = 3x - 6$
 B. $y = 3(x - 6)$
 C. $y = 3(6 - x)$
 D. $y = 6 - 3x$

Notes

LESSON 4.2

Forms of Linear Equations



CAREER SPOTLIGHT: Geological Technician

Occupation Description

Geological technicians provide support to scientists and engineers in exploring and extracting natural resources, such as oil and natural gas.

Geological technicians tend to specialize either in fieldwork and laboratory work or in office work analyzing data. However, many technicians have duties that overlap into multiple areas.

Education

Although some entry-level positions require only a high school diploma, most employers prefer applicants who have at least an associate's degree or 2 years of postsecondary training in applied science or a science-related technology. Geological technician jobs that are data intensive or otherwise highly technical may require a bachelor's degree.

Potential Employers

The largest employers of geological technicians are as follows:

Support activities for mining	23%
Oil and gas extraction	19%
Engineering services	13%
Management, scientific, and technical consulting services	5%
Management of companies and enterprises	3%

Watch a video about geological technicians:

<https://cdn.careeronestop.org/OccVids/OccupationVideos/19-4041.00.mp4>

Career Cluster

Agriculture, Food & Natural Resources

Career Pathway

Natural Resource Systems

Career Outlook

- Salary Projections:
Low-End Salary, \$28,530
Median Salary, \$51,130
High-End Salary, \$104,660
- Jobs in 2018: 16,300
- Job Projections for 2028:
17,400 (increase of 7%)

Algebra Concepts

- Compare properties of two functions represented in different ways.
- Solve problems by comparing two functions represented in different ways.

Is this a good career for me?

Geological technicians:

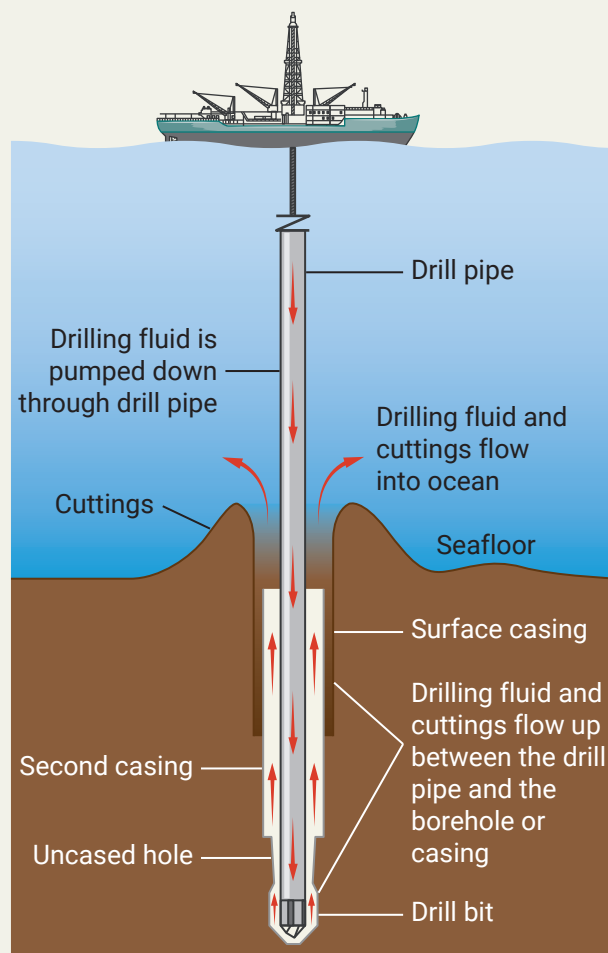
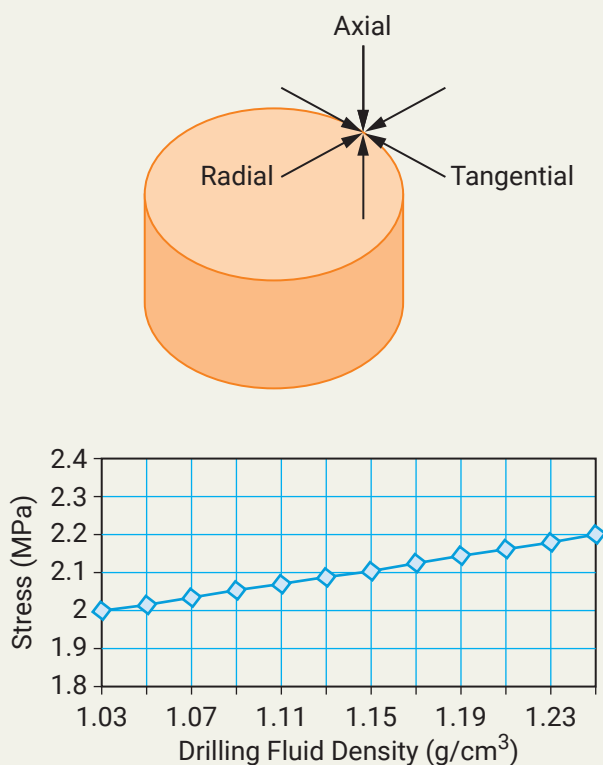
- Install and maintain laboratory and field equipment
- Gather samples such as rock, mud, and soil in the field and prepare samples for laboratory analysis
- Conduct scientific tests on samples to determine their content and characteristics

Lesson Objective

In this lesson, you will look at how a geological technician uses different representations of functions to analyze collected data.

1 Step Into the Career: Comparison of Linear Equations

When boring a hole, it is important to understand the stresses the borehole experiences so that it does not fail. A geological technician is compiling the functions for the effective stresses of a borehole 500 meters below the seafloor. The function for effective radial stress is graphed below. The effective tangential stress in megapascals is modeled by the function $S_t = -36d + 43$ for $1.03 \leq d \leq 1.25$, where d is the drilling fluid density in grams per cubic centimeter. Compare the functions.



Devise a Plan

Step 1: Determine which representation makes sense for both functions.

Step 2: Represent both functions the same way.

Step 3: Find the slope and y-intercept of both functions.

Step 4: Use the slopes and y-intercepts to compare the functions.

Walk Through the Solution

Step 1: Determine which representation makes sense for both functions.

Because the graph does not show the y -axis, graphing the y -intercept might be challenging. Instead, use points on the graph to write an equation for the effective radial stress.

Step 2: Represent both functions the same way.

Two points on the graph representing effective radial stress appear to be $(1.03, 2)$ and $(1.25, 2.2)$. Find the slope of the line.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{2.2 - 2}{1.25 - 1.03} = \frac{0.2}{0.22} \approx 0.91$$

Use either point and the slope to write an equation for the line.

$$y - y_1 = m(x - x_1)$$

Use the point-slope form of a line.

$$y - 2 = 0.91(x - 1.03)$$

Substitute 0.91 for m and $(1.03, 2)$ for (x_1, y_1) .

$$y - 2 = 0.91x - 0.94$$

Use the distributive property. Round to the nearest hundredth.

$$y = 0.91x + 1.06$$

Use the addition property of equality.

$$S_r = 0.91d + 1.06$$

Substitute d for the independent variable and S_r for the dependent variable.

So, the equation for effective tangential stress is $S_t = -36d + 43$ for $1.03 \leq d \leq 1.25$, and the equation for effective radial stress is $S_r = 0.91d + 1.06$ for $1.03 \leq d \leq 1.25$.

Step 3: Find the slope and y -intercept of both functions.

The slope of the effective tangential stress function is -36 , and the y -intercept is 43 . The slope of the effective radial stress function is 0.91 , and the y -intercept is 1.06 .

Step 4: Use the slopes and y -intercepts to compare the functions.

The slopes have opposite signs, so the functions are moving in different directions. The effective tangential stress function is decreasing while the effective radial stress function is increasing. Because the effective tangential stress function decreases from 43 while the effective radial stress function increases from 1.06 , the graphs intersect, and there is a point where the effective tangential stress equals the effective radial stress.

On the Job: Apply Comparison of Linear Equations

1. A geological technician is helping a researcher examine the effects of explosive charges inside a borehole. When an air column is above a stack of spherical charges, the pressures appear as approximated in the table. When an air column is between two stacks of spherical charges, the pressure is approximated by the function $P = 180n$, where P is the initial pressure in megapascals and n is the number of charges in each stack. Compare the functions.

**Air Column Above
One Stack of Charges**

Number of Charges in Each Stack	Initial Pressure (megapascals)
1	18
2	62
3	106
4	150

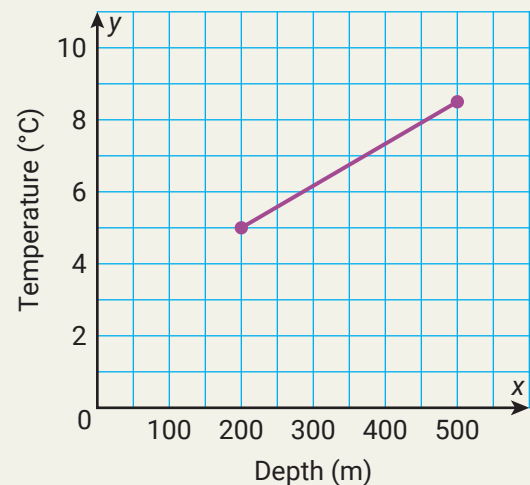
2 Step Into the Career: Problem Solving with Linear Equations

A geological technician is analyzing temperatures of boreholes in eastern Canada. At Location A, the temperatures are related to the depth of the borehole as shown in the graph. At Location B, the temperatures are related as shown in the table. At which location do you expect the greater surface temperature?

Location B

Depth (m)	Temperature (°C)
200	5.2
300	6.8
400	8.4
500	10.0

Location A



Devise a Plan

- Step 1:** Determine the characteristic of the functions that will help you answer the question.
- Step 2:** Represent both functions graphically.
- Step 3:** Make an educated guess about the answer to the question.
- Step 4:** Write an equation for Location A to determine its surface temperature.
- Step 5:** Write an equation for Location B to determine its surface temperature.

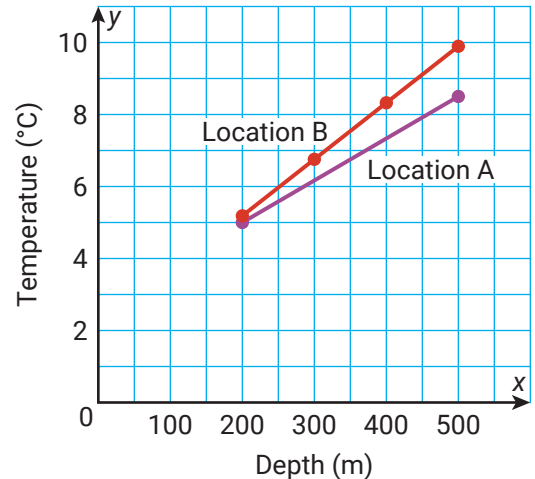
Walk Through the Solution

Step 1: Determine the characteristic of the functions that will help you answer the question.

The surface corresponds to a depth of 0 meters, so the y -intercepts will help you answer the question.

Step 2: Represent both functions graphically.

Because you need to find the y -intercepts, you want to use the graphs of the functions or the equations of the functions. The function for Location A is already graphed, so it makes sense to graph the function for Location B as well. To graph the function for Location B, plot the points (200, 5.2), (300, 6.8), (400, 8.4), and (500, 10). Then draw the line segments connecting the points.



Step 3: Make an educated guess about the answer to the question.

The graph shows that the functions are distanced from each other at $x = 500$ but close at $x = 200$. If you extend the lines to the left, they will intersect. After intersecting, the line for Location A will be above the line for Location B, which means the function for Location A has the greater y -intercept.

Step 4: Write an equation for Location A to determine its surface temperature.

One way to find the y -intercept is to write the slope-intercept form of a linear equation. That means you need to find the slope. Use points (200, 5) and (500, 8.5) to find the slope of the line for Location A.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{8.5 - 5}{500 - 200} = \frac{3.5}{300} \approx 0.012$$

Then substitute what you know into the slope-intercept form of a linear equation.

$$y = mx + b$$

Use the slope-intercept form of a line.

$$5 = 0.012(200) + b$$

Substitute 0.012 for m and (200, 5) for (x, y) .

$$5 = 2.4 + b$$

Simplify.

$$2.6 = b$$

Use the subtraction property of equality.

The y -intercept for Location A is 2.6.

Step 5: Write an equation for Location B to determine its surface temperature.

Another way to find the y-intercept is to write the point-slope form of a linear equation and then rewrite it in slope-intercept form. Use points (200, 5.2) and (500, 10) to find the slope of the line for Location B.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{10 - 5.2}{500 - 200} = \frac{4.8}{300} = 0.016$$

Then substitute what you know into the point-slope form of a linear equation.

$y - y_1 = m(x - x_1)$
 $y - 10 = 0.016(x - 500)$
 $y - 10 = 0.016x - 8$
 $y = 0.016x + 2$

Use the point-slope form of a line.
 Substitute 0.016 for m and (500, 10) for (x_1, y_1) .
 Use the distributive property.
 Use the addition property of equality.

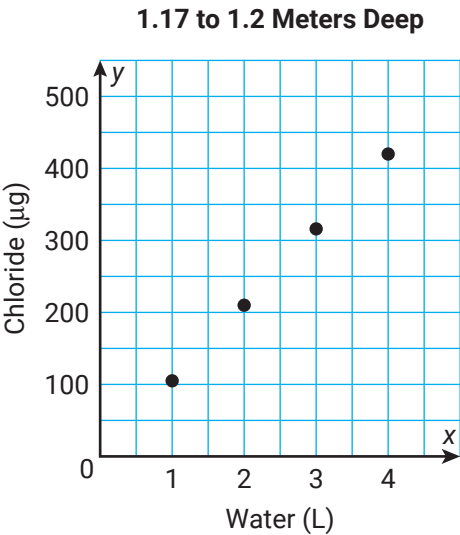
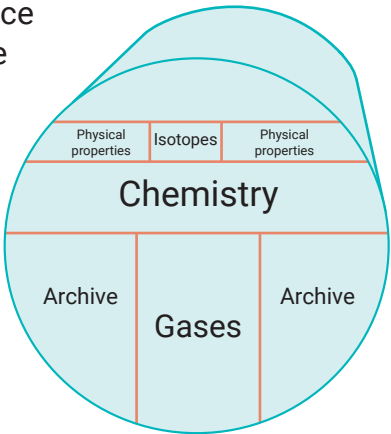
The y-intercept for Location B is 2.

Because the y-intercept for Location A is greater than the y-intercept for Location B, Location A is expected to have the greater surface temperature.

On the Job: Apply Problem Solving with Linear Equations

2. Ice core samples are taken in the shape of a cylinder. Analyzing ice core samples requires melting them. To avoid destroying a whole sample, a sample can be sectioned as shown in the diagram. Sections are analyzed for different purposes or archived.

A geological technician is analyzing data from an ice core samples from different depths. These depths represent time periods ten years apart. The sample that was 0.33 to 0.36 meters deep suggests that a liter of water at that time would have contained 111 micrograms of chloride.



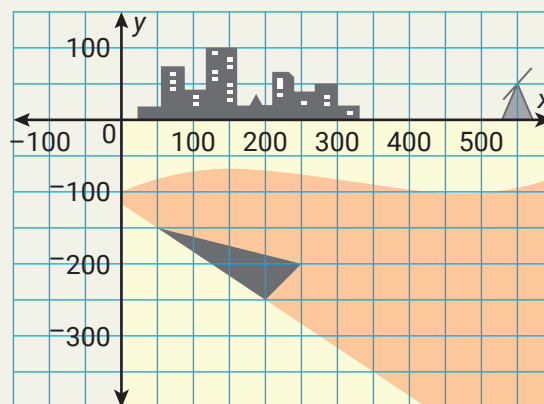
1.98 to 2.01 Meters Deep

Water (liters)	Chloride (micrograms)
2	118
4	236
6	354

Over these twenty years, did the concentration of chloride increase, decrease, or stay the same? Explain.

3 Step Into the Career: Intersecting Lines

An oil company has determined there is enough oil under a city to justify drilling a well. Because they cannot build a well in the city, they will drill at an angle from outside the city. The oil reservoir is modeled on the coordinate plane. A geological technician proposes the path of the drill follow the line $y = 0.6x - 350$ for $200 \leq x \leq 500$. Does the proposed path of the drill intersect the lines that define the reservoir? Explain what your answer means for the oil company.



Devise a Plan

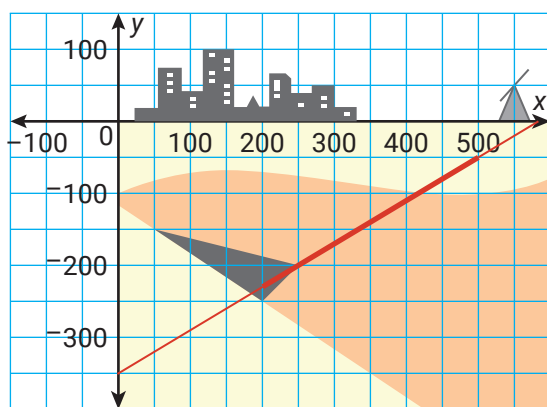
Step 1: Graph the proposed path of the drill.

Step 2: Determine whether any lines intersect.

Step 3: Reason about the needs of the oil company and whether the result of Step 2 is good or bad for the company.

Walk Through the Solution

Step 1: Graph the proposed path of the drill. Then highlight the portion of the line that falls within the boundary values of x .



Step 2: Determine whether any lines intersect.

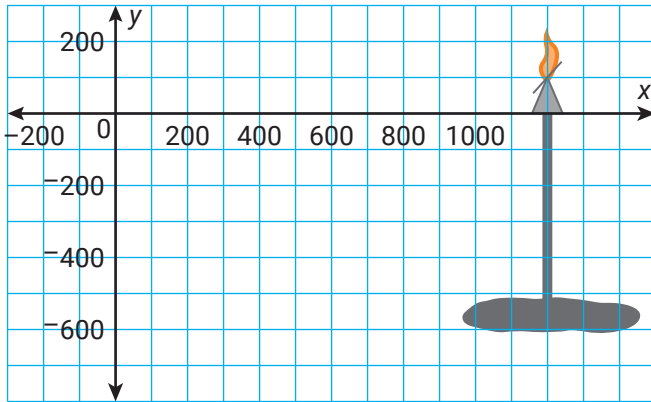
From examining the graph, the path of the drill appears to intersect one border of the oil reservoir at about $(225, -225)$.

Step 3: Reason about the needs of the oil company and whether the result of Step 2 is good or bad for the company.

The path of the drill intersects one border of the reservoir and ends before it intersects another, so the company will reach the oil.

On the Job: Apply Intersecting Lines

3. An oil well that was drilled vertically has blown, meaning it is flowing uncontrollably. The only way to stop the blowout is to drill a relief well at another location at an angle that will intersect the original well above the oil deposit. The original well extends 600 meters down from ground level, as modeled on the coordinate plane.



A geological technician suggests the diagonal portion of the relief well should follow the line $y = -\frac{1}{2}x + 200$ for $600 \leq x \leq 1200$. Will drilling this path result in a successful relief well? Explain.

Career Spotlight: Practice

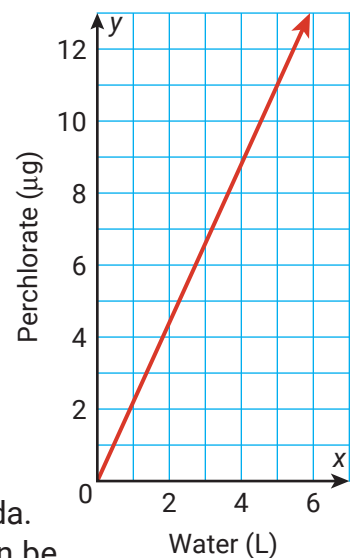
4. A geological technician who works for the Environmental Protection Agency (EPA) tested groundwater in a particular location. The graph shows the amount of perchlorate in various amounts water five years ago. In each liter of water now the amount of perchlorate is 92 micrograms. Compare the functions.

Devise a Plan

Step 1: Represent both functions the same way.

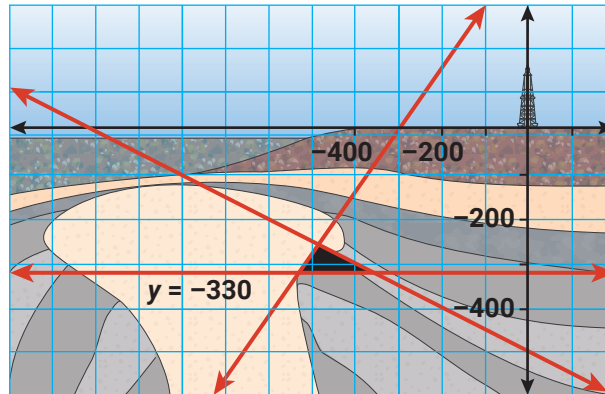
Step 2: _____ ?

Step 3: _____ ?



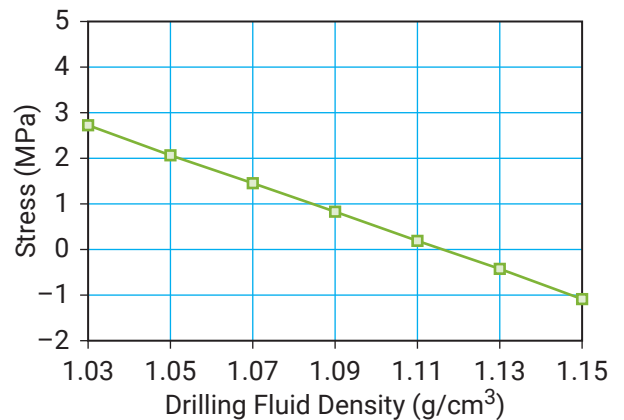
5. A geological technician is checking the temperature of a well in Nevada. For the first 1126 meters below ground surface, the temperature can be found using the equation $T = 0.0036d$, where T is in $^{\circ}\text{C}$ and d is in meters. After 1126 meters, the temperature rises 0.0376°C with each meter of descent. Does the temperature increase faster at 1000 meters below ground surface or at 2000 meters below ground surface? Explain.

6. A petroleum company wants to extract oil from beneath a salt dome. To do so, they must drill around it. The oil is bordered by the lines shown on the graph. A geological technician suggests drilling along the line $y = \frac{1}{4}x - 200$ for $-450 \leq x \leq -100$. How many of the oil's borders does this line intersect? What does that mean for the petroleum company?



Career Spotlight: Check

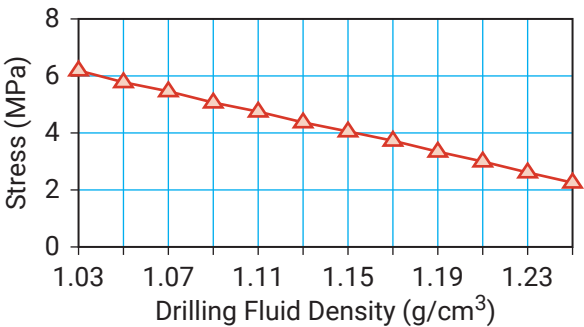
7. A geological technician is comparing the functions for the effective tangential stresses of a borehole 200 meters below the seafloor and 500 meters below the seafloor. The function for effective tangential stress at 200 meters is graphed. The effective tangential stress at 500 meters follows the function $S_t = -36d + 43$ for $1.03 \leq d \leq 1.15$, where d is the drilling fluid density. How do the functions compare?



Select all the statements that are true.

- Both functions are increasing.
- Both functions are decreasing.
- The tangential stress at 500 meters increases faster than the tangential stress at 200 meters.
- The tangential stress at 500 meters decreases faster than the tangential stress at 200 meters.
- The tangential stresses at 500 meters and at 200 meters increase at the same rate.
- The tangential stresses at 500 meters and at 200 meters decrease at the same rate.

8. A geological technician is compiling the functions for the effective stresses of a borehole 500 meters below the seafloor. The function for effective axial stress is graphed. The effective tangential stress follows the function $S_t = -36d + 43$ for $1.03 \leq d \leq 1.25$, where d is the drilling fluid density. Compare the functions.

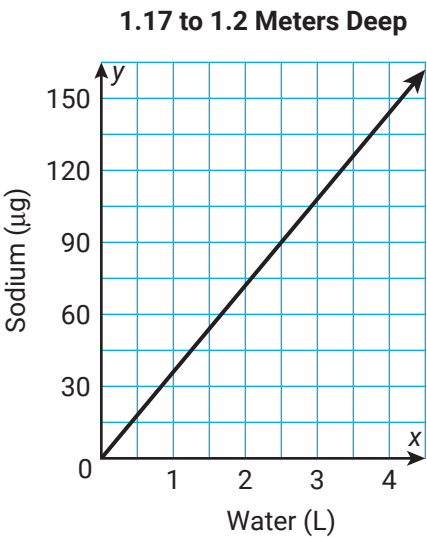


- A. Both functions are increasing, but the tangential stress is increasing faster than the axial stress.
- B. Both functions are increasing, but the axial stress is increasing faster than the tangential stress.
- C. Both functions are decreasing, but the tangential stress is decreasing faster than the axial stress.
- D. Both functions are decreasing, but the axial stress is decreasing faster than the tangential stress.

9. A geological technician is analyzing data from an ice core samples from different depths. These depths represent time periods ten years apart, with each deeper sample being ten years older. The sample that was 0.33 to 0.36 meters deep suggests that a liter of water at that time would have contained 56 micrograms of sodium.

1.98 to 2.01 Meters Deep

Water (liters)	Sodium (micrograms)
2	36
4	72
6	108



Over these twenty years, how did the concentration of sodium change?

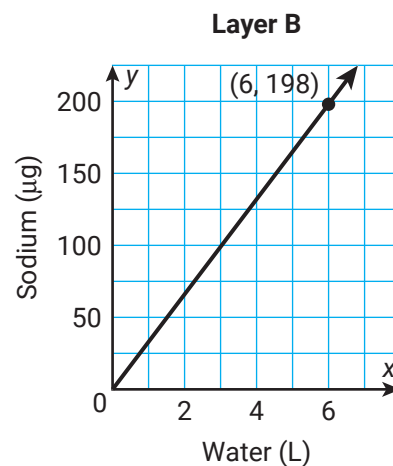
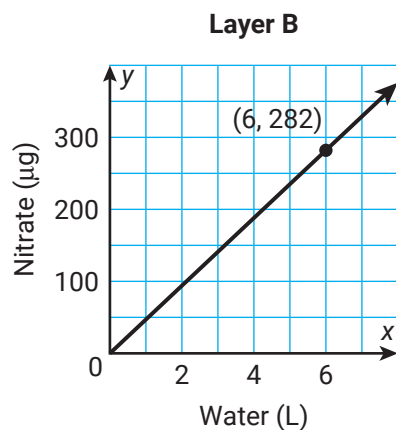
Select all the statements that are true.

- a. The concentration of sodium increased after the first ten years.
- b. The concentration of sodium decreased after the first ten years.
- c. The concentration of sodium stayed the same after the first ten years.
- d. The concentration of sodium increased after the last ten years.
- e. The concentration of sodium decreased after the last ten years.
- f. The concentration of sodium stayed the same after the last ten years.

10. A geological technician is analyzing data from an ice core sample taken in Antarctica. Layer B is 5 years older than Layer A. The sample at Layer A suggests that a liter of water at that time would have contained 58 micrograms of sulfate and 65 micrograms of nitrate.

Layer A	
Water (liters)	Sodium (micrograms)
2	66
4	132
6	198

Layer B	
Water (liters)	Sulfate (micrograms)
2	122
4	244
6	366



Select the answer from each box that makes the sentence true.

The concentration of

- a. nitrate
- b. sodium
- c. sulfate
- d. none of these

has increased after 5 years.

The concentration of

- a. nitrate
- b. sodium
- c. sulfate
- d. none of these

has decreased after 5 years.

The concentration of

- a. nitrate
- b. sodium
- c. sulfate
- d. none of these

has stayed the same after 5 years.

11. A petroleum company wants to drill toward two oil deposits from the same offshore platform. Which of these functions can the geological technician suggest?

Select all the functions that can be suggested by the geological technician.

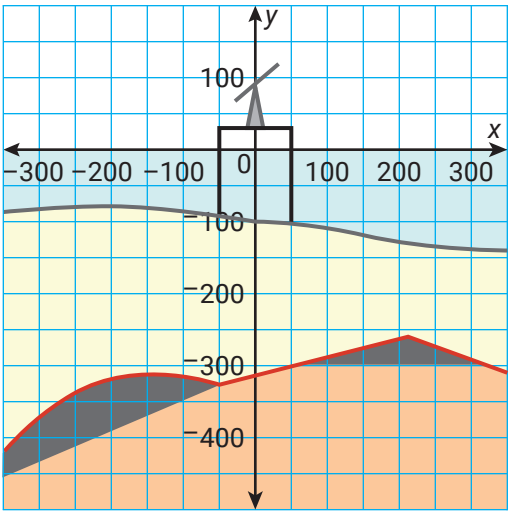
- a. $y = \frac{2}{5}x - 200$ for $-190 \leq x \leq -40$
- b. $y = \frac{2}{5}x - 250$ for $-250 \leq x \leq -50$
- c. $y = x - 150$ for $-190 \leq x \leq -40$
- d. $y = -x - 150$ for $40 \leq x \leq 140$

e.

x	y
50	-275
100	-275
150	-275
200	-275

f.

x	y
50	-200
100	-250
150	-300
200	-350



12. An oil company has determined there is enough oil under a bay to drill a well. However, it is cheaper to build an onshore oil rig and access the oil at an angle than to build an offshore oil rig and drill straight down. The oil reservoir is shown on the graph. A geological technician proposes the path of the drill follow the line $y = -0.4x - 340$ for $100 \leq x \leq 500$. Should the company drill along this path?

Select the answer from each box that makes the sentence true.

The path of the drill

- a. intersects
- b. does not intersect

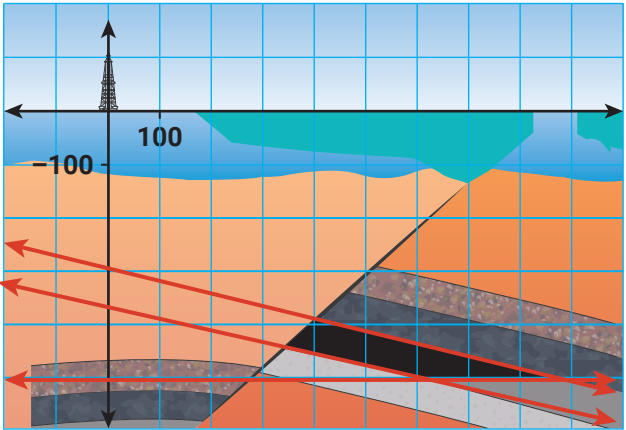
the boundaries of the reservoir

- a. at all
- b. once
- c. twice

This means the company

- a. will
- b. will not

be able to harvest oil from this reservoir using this path.



LESSON 6.2

Exponential Decay



CAREER SPOTLIGHT: Forensic Science Technician

Occupation Description

Forensic science technicians aid criminal investigations by collecting and analyzing evidence. Many technicians specialize in either crime scene investigation or laboratory analysis.

Forensic science technicians may be generalists who perform many duties or they may specialize in certain techniques and sciences. Generalist forensic science technicians, sometimes called criminalists or crime scene investigators, collect evidence at the scene of a crime and perform scientific and technical analysis.

Education

Forensic science technicians typically need at least a bachelor's degree in a natural science, such as chemistry or biology, or in forensic science. Many of those who seek to become forensic science technicians will have an undergraduate degree in the natural sciences and a master's degree in forensic science.

Potential Employers

The largest employers of forensic science technicians are as follows:

Local government, excluding education and hospitals	59%
State government, excluding education and hospitals	28%
Medical and diagnostic laboratories	4%
Testing laboratories	2%

Watch a video about forensic science technicians:

<https://cdn.careeronestop.org/OccVids/OccupationVideos/19-4092.00.mp4>

Career Cluster

Law, Public Safety, Corrections & Security

Career Pathway

Law Enforcement Services

Career Outlook

- Salary Projections:
Low-End Salary, \$35,620
Median Salary, \$59,150
High-End Salary, \$97,350
- Jobs in 2018: 16,700
- Job Projections for 2028:
19,100 (increase of 14%)

Algebra Concepts

- Apply exponents in situations where a quantity decays by a constant percent rate.
- Apply properties of exponents.

Is this a good career for me?

Forensic science technicians:

- Analyze crime scenes to determine what evidence should be collected and how
- Record observations and findings, such as the location and position of evidence
- Catalog and preserve evidence for transfer to crime labs
- Reconstruct crime scenes

Lesson Objective

In this lesson, you will look at how a forensic science technician uses exponential functions in situations where a quantity decays by a constant percent rate.

Newton's Law of Cooling

The rate at which an object cools is proportional to the difference between the temperature of the object and the temperature of the surroundings. If T is the temperature of the object at time t , T_a is the temperature of the surroundings (ambient temperature) and T_0 is the initial temperature of the object. The formula for Newton's law of cooling is as follows:

$$T(t) = (T_0 - T_a)e^{-kt} + T_a, \text{ where } k \text{ is a constant determined by the situation}$$

Half-Life

If a substance experiences exponential decay over time, there is an associated constant known as the **half-life** of the substance. This is the time it takes for half of any given amount of the substance to decay. In an exponential decay model, if the amount of the substance initially present, or peak amount, is N_0 and the half-life of the substance is h , then the amount A of the substance at time t is given by the following:

$$A(t) = N_0 e^{-0.693t/h} \text{ or } A(t) = N_0 \left(\frac{1}{2}\right)^{t/h}$$

1 Step Into the Career: Exponential Decay

Kadeem is a forensic science technician working on establishing the time line of events. One person claims that she left an office building at 3:00 p.m., just after having made her colleague a cup of coffee. In a room where the temperature was 75 °F, the temperature of the coffee at 4:30 p.m. was 120 °F. If the coffee comes out of the coffee maker at 200 °F and the value of k in this situation is 0.0128, did the woman leave at 3:00 p.m. as claimed or at 3:10 p.m., as another person claims?



Devise a Plan

Step 1: Determine the values to use in the formula for Newton's law of cooling.

Step 2: Using Newton's law of cooling, determine the temperature of the coffee at 4:30 p.m. if it was made at 3:00 p.m. and if it was made at 3:10 p.m.

Step 3: Determine what time the coffee was made using the results from Step 2.

Walk Through the Solution

Step 1: Determine the values to use in the formula for Newton's law of cooling.

The temperature of the room is 75 °F, so $T_a = 75$. The coffee comes out of the coffee maker at 200 °F, so $T_0 = 200$. The value of k for this situation is given as 0.0128.

$$\begin{aligned}T(t) &= (T_0 - T_a)e^{-kt} + T_a \\&= (200 - 75)e^{-0.0128t} + 75 \\&= 125e^{-0.0128t} + 75\end{aligned}$$

Step 2: Using Newton's law of cooling, determine the temperature of the coffee at 4:30 p.m. if it was made at 3:00 p.m. and if it was made at 3:10 p.m.

If the coffee was made at 3:00 p.m., then at 4:30 p.m., it would have been cooling for 90 minutes. However, if the coffee was made at 3:10 p.m., then it would only have been cooling for 80 minutes at 4:30 p.m.

$$\begin{aligned}T(90) &= 125e^{-0.0128 \cdot 90} + 75 \approx 115 \\T(80) &= 125e^{-0.0128 \cdot 80} + 75 \approx 120\end{aligned}$$

Step 3: Since the temperature of the coffee was 120 °F at 4:30 p.m., the coffee was made at 3:10 p.m.

The woman left at 3:10 p.m., as the other person claims.

On the Job: Apply Exponential Decay

1. At a crime scene in a house, a forensic science technician is trying to determine when a roast beef was removed from an oven in order to determine the time line of events. The roast beef was cooked to medium rare, so it was pulled from the oven with an internal temperature of 120 °F–125 °F. The temperature of a roast will increase by about 5 °F in the first five minutes after being removed from the oven before it starts to cool following Newton's law of cooling. Person A says that the roast was removed from the oven at 8:00 p.m. with an internal temperature of 125 °F, while Person B says that the roast was removed at 8:25 p.m. with an internal temperature of 120 °F.



The temperature of the house is 70 °F, and the value of k in this situation is determined to be approximately 0.01.

- a. Write the Newton's law of cooling formula that models the situation if Person A has the correct information.
- b. Write the Newton's law of cooling formula that models the situation if Person B has the correct information.
- c. What is the internal temperature of the beef at 10:00 p.m. according to each person's time line?
- d. If the internal temperature of the roast at 10:00 p.m. is about 88 °F, which person is more likely to have the correct information about the time line of events?

2 Step Into the Career: Exponential Decay

There is an investigation of whether a person trespassed into a restricted area. The area is restricted in part due to the presence of metallic mercury vapor in concentrations that are dangerous. If a person did trespass, he or she was likely exposed to mercury that would lead to blood levels of 150 micrograms per liter (mcg/L) shortly after being in the area. Two different complaints were filed on the same day describing the alleged trespassing.



The first complaint states that the person trespassed about 30 days before the complaint was filed, while the second complaint states that the person trespassed about 6 days before the complaint was filed. The person accused of trespassing claims that she did not trespass at all. Vicente is the forensic science technician in charge of finding out which account is most likely. If the half-life of mercury in the human body is approximately 50 days and a blood test administered 2 days after the complaints were made shows that the person has blood mercury levels of 95 mcg/L, which complaint is most likely? (Note that normal blood mercury levels are less than 10 mcg/L.)

Devise a Plan

Step 1: Write an exponential decay equation that models the situation.

Step 2: Use the known quantities to find the expected blood mercury levels for each account.

Step 3: Compare the expected levels to the level found in the blood test.

Walk Through the Solution

Step 1: Write an exponential decay equation that models the situation.

If t is the number of days since the alleged trespassing, then the blood test was administered at day $t + 2$. If the person trespassed, then the initial amount, N_0 , is 150. Since the half-life of mercury in humans is approximately 50 days, the exponential decay equation is:

$$A(t) = 150e^{-0.693(t+2)/50} = 150e^{(-0.693t/50) - 2 \cdot 0.693/50} = 150e^{-0.693t/50} e^{-0.02772} \approx 145.9e^{-0.01386t}$$

Step 2: Use the known quantities to find the expected blood mercury levels for each account.

If the alleged trespass took place 30 days before the complaints, then the expected level of mercury is $A(30) = 145.9e^{-0.01386 \cdot 30} \approx 96.3$ mcg/L.

If the alleged trespass took place 6 days before the complaints, then the expected level of mercury is $A(6) = 145.9e^{-0.01386 \cdot 6} \approx 134$ mcg/L.

Step 3: A blood mercury level of 95 mcg/L is close to 96.3 mcg/L but not close to either 134 mcg/L or 10 mcg/L.

It is likely that the first complaint—that the person trespassed 30 days before the complaint was filed—is correct.

On the Job: Apply Exponential Decay

2. Yubi is part of an investigation and must determine if a person received a medication as scheduled. The particular medication reaches peak blood concentration of about 12 nanograms per milliliter (ng/mL) 4 hours after being administered and has a half-life of 70 hours in the human body. She is given a blood sample that was taken 24 hours after the medication should have been administered.



Let t be the number of hours since the medication was administered.

- Write the exponential decay equation that models this situation.
- What is the concentration of the medication that Yubi should expect in the blood sample if the medication were administered on schedule?
- If the blood concentration of the medication in the sample is 10 ng/mL, was the medication likely administered as scheduled?

Career Spotlight: Practice

3. The temperature at which water boils depends on the altitude—the boiling point is lower at higher altitudes. Alys is a forensic science technician who needs to determine when a vat of water boiled over, causing an electrical short in the heating system so that the water was no longer being heated once the water boiled over. The incident occurred at an altitude of 6000 feet, where water boils at 201°F (as opposed to 212°F at sea level). One person claims that the water was about 5 minutes from boiling when he left for lunch at 11:30 a.m., while another person's account is that the water boiled over at 11:00 a.m.



The room where the water was being heated is kept at a constant 85°F , and the value of k in this situation was determined to be about 0.01.

- Write the formula for Newton's law of cooling that models the situation if the water boiled at 11:35 a.m.—that is, if the person who left for lunch at 11:30 is correct.
- Write the formula for Newton's law of cooling that models the situation if the person who claims that the water boiled at 11:00 a.m. is correct.
- What would the temperature of the water be at 1:00 p.m. if the water boiled over at 11:35 a.m.?
- What would the temperature of the water be at 1:00 p.m. if the water boiled over at 11:00 a.m.?
- If the temperature of the water at 1:30 p.m., when Alys arrived, was measured to be approximately 126°F , did the water boil over at 11:00 a.m. or after 11:30 a.m.?

4. To determine a sequence of events, Feng needs to determine which of two proposed scenarios is more likely, both involving a roast beef that was cooked to a medium-well level, which is a final internal temperature of 145°F – 150°F . This means that the roast had an internal temperature 5°F lower than its final internal temperature when it was pulled out of the oven, five minutes before it reached its final internal temperature. According to Person C, the roast was pulled out of the oven with an internal temperature of 145°F at 6:00 p.m., while according to Person D, the roast had an internal temperature of 140°F when it was pulled from the oven at 6:30 p.m. When Feng checks the temperature on the roast at 9:30 p.m., he finds that it has an internal temperature of 97°F .



If the kitchen is at a temperature of 87°F and the value of k for this situation is 0.011, which person's narrative is more likely?

Devise a Plan

Step 1: Write the formula for Newton's law of cooling for Person C's scenario.

Step 2: _____ ?

Step 3: _____ ?

Step 4: _____ ?

Step 5: _____ ?

5. An investigation needs to determine which of two accounts is credible. The main difference is the time when a medication was taken, either at 6 a.m. or at 4 p.m. on Monday. The medication in question reaches peak blood concentration of about 15 nanograms per milliliter (ng/mL) around 2 hours after being administered and has a half-life of 39 hours in the human body. A blood sample is taken at 8 a.m. on Tuesday.



Let t be the number of hours since the medication was taken.



QUICK TIP

Since the peak concentration is reached 2 hours after the medication is taken, the time variable needs to be adjusted.

- Write an exponential decay function that models the situation.
- Explain why there is only one equation needed to model the situation.
- What should the concentration be in the blood sample if the medication was taken at 6 a.m. Monday?
- What should the concentration be in the blood sample if the medication was taken at 4 p.m. Monday?
- If the concentration in the blood sample is 10 ng/mL, when was the medication likely taken?

Career Spotlight: Check

6. Veltry is working on a case of suspected lead poisoning from a commercial plant. Several people claim that the plant released gases containing lead approximately 3 weeks ago, resulting in immediate lead levels of 40 micrograms per deciliter ($\mu\text{g/dL}$) of blood. The owner of the plant claims that any release of gases containing lead was several years in the past and that no gases containing lead have been released recently. The complainants currently have lead levels ranging from 22 $\mu\text{g/dL}$ to 26 $\mu\text{g/dL}$.

If t is the amount of time since the possible lead exposure and the half-life of lead in a human body is about 32 days, select the correct equation and conclusion.

- A. $A(t) = 40e^{-0.693t}$; The plant did not release gases containing lead.
- B. $A(t) = 32e^{-0.022t}$; The plant released gases containing lead.
- C. $A(t) = 40e^{-0.022t}$; The plant released gases containing lead.
- D. $A(t) = 40e^{0.022t}$; The plant released gases containing lead.

7. There are two competing claims about the time line of the release of boiling water into an underground cooling area where the water gets cooled before being released. The site is near sea level, so the boiling water would be at a temperature of 212 °F. One claim is that the water was released at the scheduled time of 8:00 a.m. on Thursday, while another claim is that the water was not released until 4:30 p.m. on Thursday. The holding area maintains a constant temperature of 57 °F. When Nieve checks the water at 8:00 a.m. on Friday, the temperature is 172 °F.



If t represents the number of hours after the boiling water was released into the holding area and the value of k is 0.013, select all statements that are true.

- a. The water was likely released at 4:30 p.m. on Thursday
- b. The water was likely released on schedule at 8:00 a.m. on Thursday.
- c. The equation that models this situation is $T(t) = 155e^{-0.013t} + 57$.
- d. The equation that models this situation is $T(t) = 212e^{-0.013t} + 57$.
- e. The equation that models this situation is $T(t) = 155e^{0.013t} + 57$.
- f. Neither of the suggested time frames is correct.

8. There are two conflicting accounts related to when a particular medication was administered to a person. The concentration of the medication in a person's bloodstream reaches its peak of about 16 micrograms per milliliter ($\mu\text{g/mL}$) 5 days after it is taken, and the half-life of the medication is 28 days. The account of the first person is that the medication was administered at 3:00 p.m. on Monday, while the second person contends that it was administered at 3:00 p.m. on Tuesday. Akemi receives a blood sample that was taken the following Monday at 3:00 p.m.



Select the answer from each box that makes the sentence true.

An exponential decay function that models this situation is

- a. $A(t) = 16e^{-0.693(t-5)/28}$
 b. $A(t) = 16e^{-0.693t/28}$
 c. $A(t) = 16e^{-0.693(t-28)/5}$

where t is the number of days after the medication was administered. The function

simplifies to

- a. $A(t) = 16e^{-0.02475t}$
 b. $A(t) = 18.1e^{-0.02475t}$
 c. $A(t) = 18.1e^{-0.1386t}$
 d. $A(t) = 16e^{-0.1386t}$

. If the account of the first person is correct, then the

blood sample that Akemi analyzes should have a concentration of about

- a. $15.2 \mu\text{g/mL}$
 b. $13.5 \mu\text{g/mL}$

of the medication. If the account of the second person is correct, then the blood sample that

Akemi analyzes should have a concentration of about

- a. $13.8 \mu\text{g/mL}$
 b. $15.6 \mu\text{g/mL}$

of the medication.

9. A forensic science technician is establishing a time line for a crime scene by investigating the presence of a pesticide in soil. The initial amount of pesticide that was in the soil sample is estimated to be about 5 grams and to have been placed in the soil 4 hours earlier. The half-life of the pesticide is 10 hours.

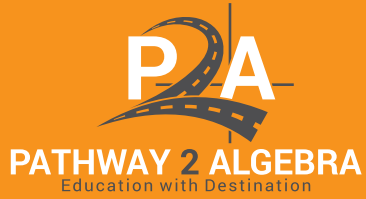


- a. Write an exponential decay function to model this situation. Let t be the number of hours since the pesticide was placed in the soil sample.
- b. Use your model to complete the table for the amount of pesticide in the soil sample. Round to the nearest hundredth of a gram.

Hour	Amount (g)
1	
2	
3	
4	

Notes

TEACHER'S EDITION



Pathway2 Algebra I



Preview Sample  NS4ed

TEACHER'S EDITION



Pathway2 Algebra I





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Pathway2Careers Algebra I

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LESSON 2.2

Solving Linear Equations with a Variable on One Side



Common Core State Standards

A-REI.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A-CED.1 Create equations and inequalities in one variable and use them to solve problems.

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Veterinarian

Veterinarians use math with a focus on calculating dosages for prescriptions and vaccines for their animal patients. This career draws from principles in medical sciences and statistics to improve the health of animal companions.

- Discuss veterinary medicine with students by reading the Career Spotlight together.
- Find local colleges and universities with a veterinary medicine program to share with students.
- Research local veterinary practices, and ask veterinarians to describe some of the things they do on a regular basis.

Video: Veterinarians

Have students watch this video, which describes the types of projects veterinarians might work on.

Lesson Objective

In this lesson, you will look at how a veterinarian writes and solves linear equations to promote the health of animal patients.

Teaching Support

1 Step Into the Career: Solving One-Step Equations

A veterinarian is prescribing medicine to a dog patient to fight infection. The suggested dosage is 1 milligram per kilogram of the dog's mass. How many pounds does the dog weigh if the dosage is 30 milligrams? Use $1 \text{ lb} = 0.454 \text{ kg}$.



Guiding Questions

- In Step 1, how can you find the dog's mass in kilograms when you know the dosage?
- In Step 2, how can you write an equation that relates weight in pounds and mass in kilograms?
- In Step 3, how can you solve an equation with a decimal multiplied by a variable?

EXTENSION In this example, the recommended dosage is 1 milligram per kilogram of body mass. Ask students to suggest how to solve a similar problem where the recommended dosage is 3 milligrams per kilogram or 5 milligrams per kilogram. What would the dog's mass be if the recommended dosage was 3 milligrams per kilogram? 5 milligrams per kilogram? (Answers: about 2.67 kg; 1.6 kg)

On the Job: Apply Solving One-Step Equations

Answers

1a. $y = 50x$

1b. 3.5 kg

Use these questions to check students' understanding.

- In 1a, how did you use 50 in writing an equation relating mass and dosage?
- In 1b, what operation did you use to solve for the mass?
- In 1b, how can you check your answer?

2 Step Into the Career: Writing and Solving Two-Step Equations

A veterinarian is treating a dog that has suffered trauma and lost blood. He estimates that the dog has about 300 milliliters of red blood cells. The veterinarian wants to give the dog a blood transfusion with whole blood that is 50% red blood cells. How much blood should the veterinarian transfuse into the dog for it to have 500 milliliters of red blood cells?



Guiding Questions

- In Step 1, how can you find the volume of red blood cells after adding the transfusion?
- In Step 1, how can you find the volume of red blood cells in the transfusion itself?
- In Step 3, what is the order of operations when solving a two-step equation?

EXTENSION Consider a similar problem where a dog who needs blood has 2 liters of blood, and the veterinarian wants to give a transfusion of blood so that 25% of the dog's blood is red blood cells. How could you write an equation for the volume of red blood cells after the transfusion? What volume of blood is required to be added for the dog to have 550 milliliters of red blood cells? (Answers: $y = 0.25(x + 2000)$; 200 mL)

On the Job: Apply Writing and Solving Two-Step Equations

Answers

2a. $y = 0.6x + 9$

2b. 10 mL

Use these questions to check students' understanding.

- In 2a, how did you write 60% as a decimal?
- In 2b, what volume of red blood cells did you substitute for y ?
- In 2b, what order did you perform the operations to solve for x ?

3 Step Into the Career: Using the Distributive Property

A veterinarian has 400 milliliters of a solution that is 10% medicine. Her patient needs 55 milliliters of the actual medicine. How much more 10% solution does she need for her patient?



Guiding Questions

- In Step 1, how can you write an expression for the total amount of medicine in a 10% solution?
- In Step 3, how can you solve the equation without using the distributive property? Which method do you prefer?

EXTENSION Consider a similar problem where a dog needs medicine that comes in a 3% solution. The veterinarian has a container with 750 milliliters of solution, and the dog needs to get 15 milliliters of medicine. How much should be left in the container after treating the dog? (Answer: 250 mL)

On the Job: Apply Using the Distributive Property

Answers

3a. $y = 15(6(x + 4))$ or $y = 90(x + 4)$

3b. $x = 1$; The container has enough to feed the kittens 1 more time today.

Use these questions to check students' understanding.

- In 3a, what expression did you write for the number of feedings? (Possible answer: $6(x + 4)$)
- In 3b, what volume of formula did you substitute for y ? (450)
- In 3b, how did you solve for x ? (Possible answer: Used the distributive property, subtracted, and then divided.)

Career Spotlight: Practice

Solution Steps for Exercises 4–8

These steps will help guide students in solving these practice exercises.

Exercise 4

Answer

4. 15 kg

Devise a Plan

Possible plan:

Step 1: Write an equation relating the weight of the dog and the dosage.

Step 2: Substitute values from the problem.

Step 3: Solve and interpret the solution.

Solution Steps

- Let x be the mass of the dog and y be the dosage. Write an equation. ($y = 140x$)
- Substitute 2100 for y . ($2100 = 140x$)
- Divide both sides by 140 to solve for x . ($15 = x$)

Exercise 5

Answers

5a. $y = 8x$

5b. 0.85 milligram

Solution Steps

- Write an equation in the form $y = ax$, where x is the dosage. ($y = 8x$)
- Substitute the given mass for y . ($6.8 = 8x$)
- Divide both sides of the equation by the coefficient of x to solve the problem. ($0.85 = x$)

Exercise 6

Answers

6a. 15 mL

6b. $0.04(x + 300)$

6c. $15 = 0.04(x + 300)$; 75 mL

Solution Steps

- Multiply the volume of the starting solution by its percent. ($0.05(300 \text{ mL}) = 15 \text{ mL}$)
- Multiply the volume of the mixture by an expression for the total amount of fluid in the mixture. ($0.04(x + 300)$)
- Set expressions for the amount of medicine equal to each other. ($15 = 0.04(x + 300)$)
- Use the order of operations for solving equations. Divide first to isolate the parentheses. ($15 \div 0.04 = 0.04(x + 300) \div 0.04$; $375 = x + 300$; $375 - 300 = x + 300 - 300$; $75 = x$)

Exercise 7

Answer

7. 5 L; Write and solve an equation for the amount of blood x .

$$8 = 0.4x + 6$$

$$8 - 6 = 0.4x + 6 - 6$$

$$2 = 0.4x$$

$$\frac{2}{0.4} = \frac{0.4x}{0.4}$$

$$5 = x$$

Solution Steps

- Let x be the amount of blood the horse needs. Write an equation for the volume of red blood cells the horse will have after the transfusion. ($8 = 0.4x + 6$)
- Use the order of operations for solving equations. Subtract first, and then divide. ($8 - 6 = 0.4x + 6 - 6$; $2 = 0.4x$; $2 \div 0.4 = 0.4x \div 0.4$; $5 = x$)

Exercise 8

Answer

8. 65 calories

Solution Steps

- Let x be the number of calories in each serving of treats. Write an equation for the number of calories the dog should eat each day. ($1240 = 2(x + 555)$)
- Solve by using the distributive property. ($1240 = 2x + 1110$; $130 = 2x$; $65 = x$)

Career Spotlight: Check

Tips for Completing Exercises 9–16

These tips will help students in solving these exercises and similar assessment items.

Exercise 9

Answer

9. a. $y = 3x$, a. x , a. one tablet

Tip Encourage students to make a table of values to check their answers for reasonableness. The table should have columns for the mass of the dog, the suggested dosage, and the corresponding number of tablets. Remind them that one-half tablet equals 25 milligrams.

Exercise 10

Answer

10. A

Tip Encourage students to use rounding to check their answers for reasonableness.

Exercise 11

Answer

11. a, d, f

Tip Encourage students to choose the correct equation first. Remind them that only one of the first four choices is correct but any number of the last three choices could be correct.

Exercise 12

Answers

12a. $y = 30x + 70$

12b. no; The dog needs 1270 mL of IV fluids.

Tip Encourage students to first translate the phrase “70 milliliters more than 30 milliliters per kilogram” to an algebraic expression.

Exercise 13

Answer

13. C

Tip Encourage students to discuss what it means to have a 0%, 2%, or 5% solution of medicine.

Exercise 14

Answer

14. 2.3 ounces

Tip Encourage students to first write a one-step equation for y milligrams of toxins in x ounces of chocolate.

Exercise 15

Answer

15. b. $y = 4.5x$, c. 30, a. is, b. $x \approx 6.667$, a. more than

Tip Encourage students to first find the equation that describes the situation and then choose answers for the other parts as they solve. Have them check their answer by substituting 6.5 for x in the equation.

Exercise 16

Answer

16. B

Tip Encourage students to discuss how they should represent the weight a kitten should gain. Remind them that x represents the age of a kitten in weeks but that y is the weight gained since the kitten turned 6 weeks old.

LESSON 4.2

Forms of Linear Equations



Common Core State Standards

F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Geological Technician

Geological technicians provide support to scientists and engineers in exploring and extracting natural resources, such as oil and natural gas. Geological technicians collect, compile, and analyze data. They rely on a strong knowledge of arithmetic, algebra, geometry, calculus, and statistics.

- Discuss geological technology with students by reading the Career Spotlight together.
- Find local community colleges and technical institutes with a program in geosciences, petroleum, mining, or a related technology to share with students.
- Research local companies that employ geological technicians, and ask what they do for the companies.

Video: Geological Technicians

Have students watch this video, which describes the types of projects geological technicians might work on.

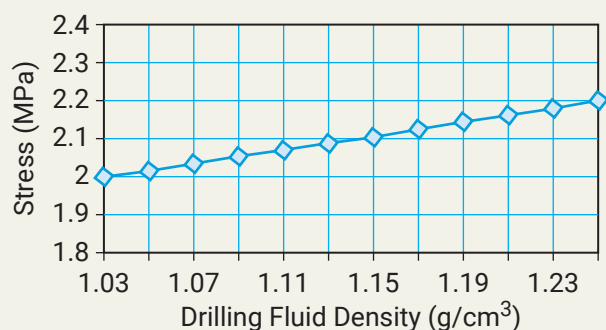
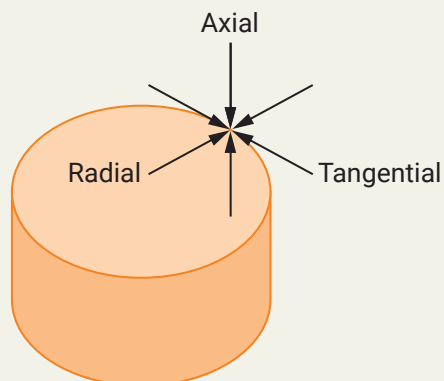
Lesson Objective

In this lesson, you will look at how a geological technician uses different representations of functions to analyze collected data.

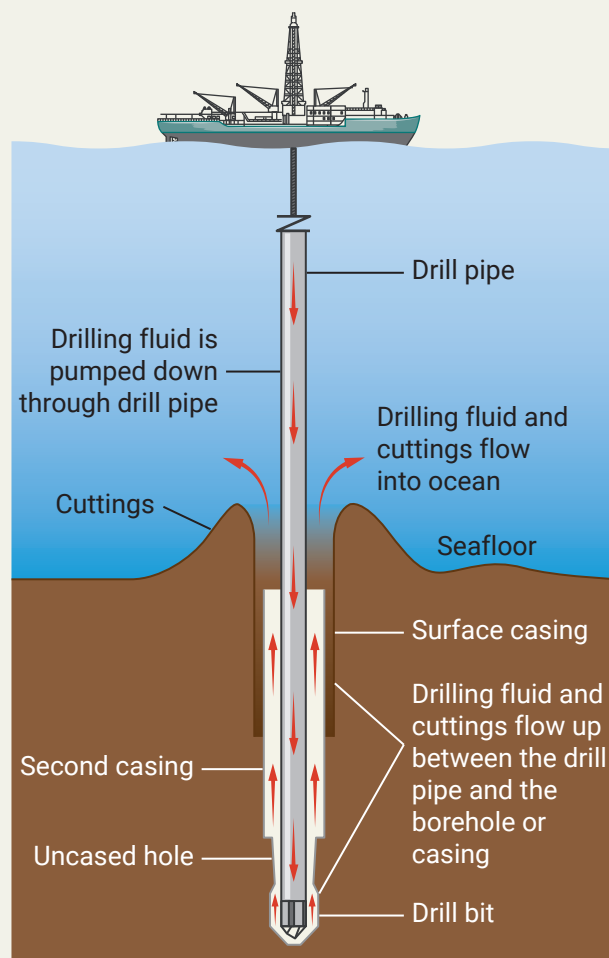
Teaching Support

1 Step Into the Career: Comparison of Linear Equations

When boring a hole, it is important to understand the stresses the borehole experiences so that it does not fail. A geological technician is compiling the functions for the effective stresses of a borehole 500 meters below the seafloor. The function for effective radial stress is graphed below. The effective tangential stress in megapascals is modeled by the function $S_t = -36d + 43$ for $1.03 \leq d \leq 1.25$, where d is the drilling fluid density in grams per cubic centimeter. Compare the functions.



The abbreviation MPa stands for megapascal.



Guiding Questions

- Does the y -axis appear in the graph?
- Before you compare the functions, does it make more sense to graph both functions, use equations for both functions, use a table for both functions, or describe both functions verbally?
- What form of linear equation is the function for effective tangential stress?
- In Step 1, after you find the slope, how can you use the point-slope form of a line to write an equation in slope-intercept form? Can you use only slope-intercept form to write an equation for the line?

ENGLISH LANGUAGE LEARNERS Some students may be familiar with the words *radius* and *tangent* as they relate to a circle. For students not familiar, it may be helpful to draw a diagram. The direction of *radial* stress follows the *radius* of the circular pipe, and the direction of *tangential* stress follows the *tangent* of the circular pipe. The tangential stress is perpendicular to the radial stress just as a tangent is perpendicular to a radius.

On the Job: Apply Comparison of Linear Equations

Answer

1. Possible answer: The slopes are both positive, so both functions are increasing. The function for the pressure when the air column is between the charges has a greater slope, so the pressure increases faster with each added charge. The y -intercept for the pressure when the air column is above the charges is negative, but all values of P for $n > 0$ are less than the values in the other function, so the lines intersect between $n = 0$ and $n = 1$. Because charges must be whole numbers, the pressure when the air column is between the charges is always greater than the pressure when the air column is above the charges.

Use these questions to check students' understanding.

- Before you compared the functions, did it make more sense to graph both functions, use equations for both functions, use a table for both functions, or describe both functions verbally?
- How did you use the information in the table to calculate the slope of the line?
- How did you know which line is steeper?
- Do the lines intersect? If so, about where?
- For what number of charges is one function greater than the other?

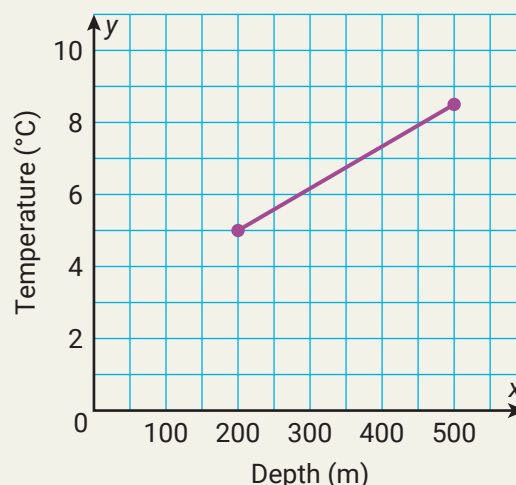
2 Step Into the Career: Problem Solving with Linear Equations

A geological technician is analyzing temperatures of boreholes in eastern Canada. At Location A, the temperatures are related to the depth of the borehole as shown in the graph. At Location B, the temperatures are related as shown in the table. At which location do you expect the greater surface temperature?

Location B

Depth (m)	Temperature ($^{\circ}\text{C}$)
200	5.2
300	6.8
400	8.4
500	10.0

Location A



Guiding Questions

- What is the depth of the hole at the surface?
- Before you compare the functions, does it make more sense to graph both functions, use equations for both functions, use a table for both functions, or describe both functions verbally?
- In Step 3, in which direction should you extend the graph?
- Do the graphs intersect? If so, which graph will be higher when they reach the y-intercept?

On the Job: Apply Problem Solving with Linear Equations

Answer

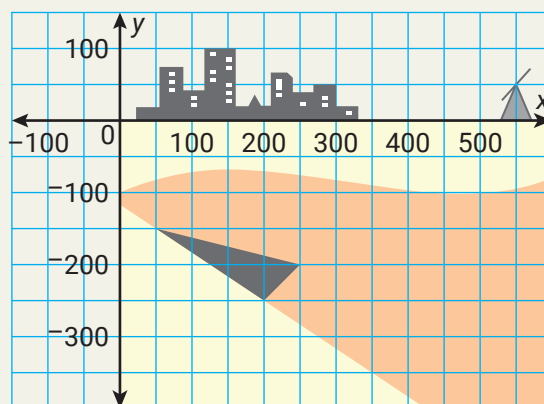
2. The concentration of chloride increased over these twenty years. The deepest sample had a concentration of 59 micrograms per liter; the middle, between 100 and 110 micrograms per liter; and the shallowest, 111 micrograms per liter.

Use these questions to check students' understanding.

- Determine the characteristic of the functions that will help you answer the question. (slope)
- Before you compare the functions, does it make more sense to graph all functions, use equations for all functions, use a table for all functions, or describe all functions verbally?
- Do the samples get older or younger as their depth increases? (older)
- What is the unit for the slope of these functions? With that in mind, what else can the slope be called? (unit rate)

3 Step Into the Career: Intersecting Lines

An oil company has determined there is enough oil under a city to justify drilling a well. Because they cannot build a well in the city, they will drill at an angle from outside the city. The oil reservoir is modeled on the coordinate plane. A geological technician proposes the path of the drill follow the line $y = 0.6x - 350$ for $200 \leq x \leq 500$. Does the proposed path of the drill intersect the lines that define the reservoir? Explain what your answer means for the oil company.



Drills cannot make sharp angles, so a curve would connect the oil well to the proposed path. For this reason, the path does not go all the way to the vertical line where the oil well is.

Guiding Questions

- Why are the values of x important?
- What would happen if the drill stopped at $x = 300$? if it stopped at $x = 150$?

DIFFERENTIATION: ENRICHMENT The term for drilling straight down and then angling away is *directional drilling*. Drills cannot make sharp angles but instead can curve, as if drilling around a circle. The conventional radius of a turn is 2800 feet. If the angle from the vertical portion of the well to the diagonal portion is 59° , how long is the curve that connects the oil well to the proposed path? Remind students that the formula for circumference of a circle is $C = 2\pi r$ and that there are 360° in a circle. As a hint, tell students they can use a proportion to determine the length of the curve.

(Answer: about 2883 ft)

On the Job: Apply Intersecting Lines

Answer

3. yes; The diagonal portion of the relief well will intersect the original well at $(1200, -400)$, which is safely above the oil deposit.

Use these questions to check students' understanding.

- What must happen for the relief well to be successful?
- Does it make more sense to graph the function, use an equation for the function, use a table for the function, or describe the function verbally?

Career Spotlight: Practice

Solution Steps for Exercises 4–6

These steps will help guide students in solving these practice exercises.

Exercise 4

Answer

4. Possible answer: Both functions are increasing, and both functions have y-intercepts of 0. The function representing the perchlorate in the groundwater now is increasing faster than the function representing that information from five years ago.

Devise a Plan

Possible plan:

Step 1: Represent both functions the same way.

Step 2: Find the slope and y-intercept of both functions.

Step 3: Use the slopes and y-intercepts to compare the functions.

Solution Steps

- Write an equation for the graph. ($y = 2.2x$)
- Write an equation for the verbal description. ($y = 92x$)
- Compare the slopes and y-intercepts. (Both functions have positive slopes. The verbal description has a greater slope. Both functions have y-intercept of 0.)

Exercise 5

Answer

5. The temperature increases faster at 2000 meters below ground surface because the slope after 1126 meters is greater than the slope before 1126 meters.

Solution Steps

- Determine the characteristic of the functions that will help you answer the question. (slope)
- Determine the slope for the first 1126 meters. ($0.0036\text{ }^{\circ}\text{C/m}$)
- Determine the slope after 1126 meters. ($0.0376\text{ }^{\circ}\text{C/m}$)

Exercise 6

Answer

6. The proposed line intersects the oil's borders once, so the pipe will end inside the oil deposit and allow the oil company to harvest the oil.

Solution Steps

- Graph the proposed path of the drill, and determine whether it intersects any borders of the oil. (It intersects one border.)
- Reason about the needs of the oil company and whether intersecting one border is good or bad for the company. (Crossing one border means the pipe ends in the oil. Crossing zero or two borders means the drill ends in a layer that does not contain oil.)

Career Spotlight: Check

Tips for Completing Exercises 7–12

These tips will help students in solving these exercises and similar assessment items.

Exercise 7

Answer

7. b, d

Tip Encourage students to analyze the answer options, which focus on the slopes of the functions.

Exercise 8

Answer

8. C

Tip Encourage students to analyze the answer options, which focus on the slopes of the functions.

Exercise 9

Answer

9. a, d

Tip Remind students that concentrations are unit rates, or slopes. Encourage students to organize the information in order of age.

Exercise 10

Answer

10. a. nitrate, c. sulfate, b. sodium

Tip Remind students that concentrations are unit rates, or slopes. Encourage students to organize the information in order of age, perhaps by making a table of concentrations with column heads Older and Newer.

Exercise 11

Answer

11. b, c, d, e

Tip Remind students of the criteria of a good well—that it enters the oil deposit but does not exit. Encourage students to graph the lines and pay particular attention to the limits on x .

Exercise 12

Answer

12. a. intersects, c. twice, b. will not

Tip Encourage students to graph the proposed path of the drill.

Notes

LESSON 6.2

Exponential Decay



Common Core State Standards

F-LE.1c Recognize situations in which a quantity decays by a constant percent rate per unit interval relative to another.

F-IF.8b Use the properties of exponents to interpret expressions for exponential functions.

Mathematical Practices 1, 2, 4

CAREER SPOTLIGHT: Forensic Science Technician

Forensic science technicians use science and math to analyze physical evidence related to criminal investigations. They may work in laboratories, at crime scenes, or both. At crime scenes, forensic science technicians may analyze evidence using chemical, biological, and microscopic methods. In laboratories, they use equipment such as microscopes and computers.

- Discuss forensic science technician work with students by reading the Career Spotlight together.
- Find local colleges and universities with a forensic science or related program to share with students.
- Research local government agencies or laboratories that employ forensic science technicians, and ask what they do.

Video: Forensic Science Technicians

Have students watch this video, which describes the types of work forensic science technicians might do.

Lesson Objective

In this lesson, you will look at how a forensic science technician uses exponential functions in situations where a quantity decays by a constant percent rate.

Teaching Support

1 Step Into the Career: Exponential Decay

Kadeem is a forensic science technician working on establishing the time line of events. One person claims that she left an office building at 3:00 p.m., just after having made her colleague a cup of coffee. In a room where the temperature was 75 °F, the temperature of the coffee at 4:30 p.m. was 120 °F. If the coffee comes out of the coffee maker at 200 °F and the value of k in this situation is 0.0128, did the woman leave at 3:00 p.m. as claimed or at 3:10 p.m., as another person claims?



Guiding Questions

- In Step 1, how was the temperature of the room used in Newton's law of cooling?
- In Step 1, how was the initial temperature of the coffee used in Newton's law of cooling?
- In Step 2, how are the different times used?

DIFFERENTIATION: ENRICHMENT Tell students that the optimal temperature to drink a cup of coffee is 155 °F–175 °F. Have students discuss how they can determine the time after brewing when the coffee is at the optimal drinking temperature.

On the Job: Apply Exponential Decay

Answers

1a. $T(t) = (130 - 70)e^{-0.01t} + 70 = 60e^{-0.01t} + 70$, where t is the number of minutes past 8:05 p.m.

1b. $T(t) = (125 - 70)e^{-0.01t} + 70 = 55e^{-0.01t} + 70$, where t is the number of minutes past 8:30 p.m.

1c. Person A: about 89 °F, Person B: about 92 °F

1d. Person A

Use these questions to check students' understanding.

- In 1a, what values did you use for T_a , T_0 , and the time at which $t = 0$?
- In 1b, what values did you use for T_a , T_0 , and the time at which $t = 0$?
- In 1c, what value of t did you use for each person's time line? Why are two different values of t needed?

2 Step Into the Career: Exponential Decay

There is an investigation of whether a person trespassed into a restricted area. The area is restricted in part due to the presence of metallic mercury vapor in concentrations that are dangerous. If a person did trespass, he or she was likely exposed to mercury that would lead to blood levels of 150 micrograms per liter (mcg/L) shortly after being in the area. Two different complaints were filed on the same day describing the alleged trespassing.



The first complaint states that the person trespassed about 30 days before the complaint was filed, while the second complaint states that the person trespassed about 6 days before the complaint was filed. The person accused of trespassing claims that she did not trespass at all. Vicente is the forensic science technician in charge of finding out which account is most likely. If the half-life of mercury in the human body is approximately 50 days and a blood test administered 2 days after the complaints were made shows that the person has blood mercury levels of 95 mcg/L, which complaint is most likely? (Note that normal blood mercury levels are less than 10 mcg/L.)

Guiding Questions

- In Step 1, why was $t + 2$ substituted when setting up the exponential decay equation?
- In Step 3, why does it make sense that the blood mercury level would be higher for a more recent trespass?

DIFFERENTIATION: ENRICHMENT How would the equation be affected if the blood sample had been taken one week after the complaints? Would the expected blood mercury levels be higher or lower? (Answers: Instead of using $t + 2$ in the exponent, use $t + 7$ in the exponent. The expected blood mercury levels would be lower because more time had passed)

On the Job: Apply Exponential Decay

Answers

2a. $A(t) = 12e^{-0.693(t-4)/70} \circ 12.48e^{-0.0099t}$

2b. about 9.8 ng/mL

2c. yes

Use these questions to check students' understanding.

- In 2a, what values did you use for N_0 and h in the exponential decay function?
- In 2a, how did you account for the medication reaching peak levels 4 hours after being administered?
- In 2b, what value for t did you use to determine what the expected amount should be?

Career Spotlight: Practice

Solution Steps for Exercises 3–5

These steps will help guide students in solving these practice exercises.

Exercise 3

Answers

3a. $T(t) = (201 - 85)e^{-0.01t} + 85 = 116e^{-0.01t} + 85$, where t is the number of minutes past 11:35 a.m.

3b. $T(t) = (201 - 85)e^{-0.01t} + 85 = 116e^{-0.01t} + 85$, where t is the number of minutes past 11:00 a.m.

3c. about 135 °F

3d. about 120 °F

3e. after 11:30 a.m.

Solution Steps

- Use the known values of T_a and T_0 to write Newton's law of cooling, where t is the number of minutes after 11:35 a.m. ($T(t) = (201 - 85)e^{-0.01t} + 85 = 116e^{-0.01t} + 85$)
- Use the known values of T_a and T_0 to write Newton's law of cooling, where t is the number of minutes after 11:00 a.m. ($T(t) = (201 - 85)e^{-0.01t} + 85 = 116e^{-0.01t} + 85$)
- Determine the number of minutes between 11:35 a.m. and 1:00 p.m. Use this value for t to find the temperature at 1:00 p.m. if the water boiled over at 11:35 a.m. (85 minutes, about 135 °F)
- Determine the number of minutes between 11:00 a.m. and 1:00 p.m. Use this value for t to find the temperature at 1:00 p.m. if the water boiled over at 11:00 a.m. (120 minutes, about 120 °F)
- Determine the temperature of the water at 1:30 p.m. in the two scenarios to find which scenario is more likely. (111 °F and 122 °F, boiled over after 11:30 a.m.)

Exercise 4

Answer

4. Person D

Devise a Plan

Possible plan:

Step 1: Write Newton's law of cooling formula for Person C's scenario.

Step 2: Write Newton's law of cooling formula for Person D's scenario.

Step 3: Determine the internal temperature of the roast at 9:30 p.m. if Person C's scenario is correct.

Step 4: Determine the internal temperature of the roast at 9:30 p.m. if Person D's scenario is correct.

Step 5: Answer the question based on the answers to Steps 3 and 4.

Solution Steps

- Write Newton's law of cooling formula for Person C's scenario.
($T(t) = (150 - 87)e^{-0.011t} + 87 = 63e^{-0.011t} + 87$, where t is the number of minutes after 6:05 p.m.)
- Write Newton's law of cooling formula for Person D's scenario.
($T(t) = (145 - 87)e^{-0.011t} + 87 = 58e^{-0.011t} + 87$, where t is the number of minutes after 6:35 p.m.)
- Determine the internal temperature of the roast at 9:30 p.m. if Person C's scenario is correct. (about 93.6 °F)
- Determine the internal temperature of the roast at 9:30 p.m. if Person D's scenario is correct. (about 95.5 °F)
- Answer the question that was asked. (Person D's narrative)

Exercise 5

Answers

5a. $A(t) = 15e^{-0.693(t-2)/39} \circ 15.54e^{-0.0178t}$

- 5b. None of the values in the exponential decay function—the peak concentration, the time that the peak concentration occurs, or the half-life of the medication—are affected by the time between the medication being taken and the time of the blood sample, which is the value of the variable t .

5c. about 9.8 ng/mL

5d. about 11.3 ng/mL

5e. at 6 a.m. on Monday

Solution Steps

- Use the given values of N_0 and h to write the exponential decay function.
($A(t) = 15e^{-0.693(t-2)/39} \circ 15.54e^{-0.0178t}$)
- Reason that the values in the exponential decay function depend only on attributes of the medication, not on the time between the medication being taken and the blood sample.
- Determine the number of hours between 6:00 a.m. Monday and 8:00 a.m. Tuesday. Use this value for t to find the concentration if the medication was taken at 6:00 a.m. Monday. (26 hours, about 9.8 ng/mL)
- Determine the number of hours between 4:00 p.m. Monday and 8:00 a.m. Tuesday. Use this value for t to find the concentration if the medication was taken at 4:00 p.m. Monday. (16 hours, about 11.3 ng/mL)
- Compare the actual concentration of the medication to the values from the model. Determine which value is closer to the actual concentration to decide when the medication was likely taken. (10 ng/mL is closer to 9.8 ng/mL than to 11.3 ng/mL, 6 a.m. Monday)

Career Spotlight: Check

Tips for Completing Exercises 6–9

These tips will help students in solving these exercises and similar assessment items.

Exercise 6

Answer

6. C

Tip Encourage students to examine the answer choices after reading the problem to eliminate choices that can easily be identified as incorrect. For example, since lead levels in humans decrease following an exponential decay model, the exponent of e will be negative, so choice D can be eliminated.

Exercise 7

Answer

7. b, c

Tip Encourage students to determine and find the information that is needed to select the true statements. For example, students should find the difference between the initial temperature of the water and the ambient temperature of the surroundings in order to select the correct equation.

Exercise 8

Answer

8. a. $A(t) = 16e^{-0.693(t-5)/28}$, b. $A(t) = 18.1e^{-0.02475t}$, a. $15.2 \mu\text{g/mL}$, b. $15.6 \mu\text{g/mL}$

Tip Encourage students to check their answers for reasonableness by reading the entire problem after they have chosen their answers. For example, the concentration of the medication in the blood sample for the third blank should be less than the concentration for the fourth blank.

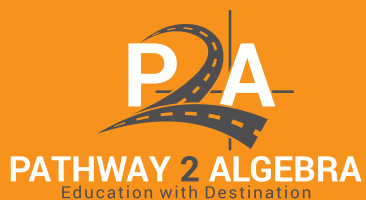
Exercise 9

Answers

9a. $A(t) = 5\left(\frac{1}{2}\right)^{t/10}$, or $A(t) = 5e^{-0.0693t}$

9b. 4.67; 4.35; 4.06; 3.79

Tip Encourage students to check that their answers in the table are reasonable. The difference in the amounts between hours should be decreasing since the model is an exponential decay. Because the half-life is 10 hours, the amounts in the table should be more than 2.5 grams.



Pathway2 Algebra I



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